Motion in One Dimension

We all know what the distance between two objects is...

So what is it?
What is distance?
What is length?

ALSO - you can't use the words "distance" or "length" in your definition; that would be cheating.

Distance

As you can see from your efforts, it is impossible to define distance.

Distance is a fundamental part of nature. It is so fundamental that it's impossible to define. Everyone knows what distance is, but no one can really say what it is.

However, distances can be compared.

Distance

We can compare the distance between two objects to the distance between two other objects.

For convenience, we create standard distances so that we can easily make comparisons... and tell someone else about them.

We will be using the meter as our unit for measuring distance. It's just that it's been accepted as a universal standard, so everyone knows what it is.

This doesn't define distance, but it allows us to work with it.
Distance
We'll be using meter as our standard for measuring distance.
The symbol for distance is "d".
And the unit for the meter is "m".

\[ d = 0.2 \text{ m} \]

Time
Similarly, everyone knows what time is...
But try defining it, what is time?
Remember you can't use the word "time"
or an equivalent to the word "time", in your definition.

Time
Like distance, time is a fundamental aspect of nature.
It is so fundamental that it's impossible to define. Everyone knows what time is, but no one can really say what it is...
However, like distances, times can be compared.

Time
We can say that in the time it took to run around the track, the second hand of my watch went around once...so my run took 60 seconds. When we compare the time between two events to the time between two other events, we are measuring time.

This doesn't define time, but it allows us to work with it.

Speed
Speed is defined as the distance traveled divided by the time it took to travel that distance.

\[ \text{speed} = \frac{\text{distance}}{\text{time}} \]

\[ s = \frac{d}{t} \]

Speed is not a fundamental aspect of nature, it is the ratio of two things that are.
1. A car travels at a constant speed of 10 m/s. This means the car:
   A. increases its speed by 10 m every second.
   B. decreases its speed by 10 m every second.
   C. moves with an acceleration of 10 meters every second.
   D. moves 10 meters every second.

2. A rabbit runs a distance of 60 meters in 20 s; what is the speed of the rabbit?

3. An airplane on a runway can cover 500 m in 10 s; what is the airplane's average speed?

4. A car travels at a speed of 40 m/s for 4.0 s; what is the distance traveled by the car?

5. You travel at a speed of 20 m/s for 6.0 s; what distance have you moved?

Speed

The units of speed can be seen by substituting the units for distance and time into the equation

\[ s = \frac{d}{t} \]

meters
second

\[ \frac{m}{s} = \text{We read this unit as "meters per second"} \]
6. You travel at a constant speed of 20 m/s; how much time does it take you to travel a distance of 120m?

7. You travel at a constant speed of 30 m/s; how much time does it take you to travel a distance of 150m?

Position and Reference Frames

Speed, distance and time didn’t require us to define where we started and where we ended up. They just measure how far we traveled and how long it took to travel that far.

However, much of physics is about knowing where something is and how its position changes with time.

To define position we have to use a reference frame.

Reference Frame Activity

Send a volunteer out of the classroom to wait for further instructions.

Place an object somewhere in your classroom. Write specific directions for someone to be able to locate the object.

Write them in a way that allows you to hand them to someone who can then follow them to the object.

Remember: you can’t tell them the name of something your object is near just how they have to move to get to it. For instance ‘walk to the SmartBoard’ is not a specific direction.

Test your directions out on your classmate. (who is hopefully still in the hallway)
Reference Frame Activity - Reflection

In your groups, make a list of the things you needed to include in your directions in order to successfully locate the object in the room.

As a class, discuss your findings.

Results - Reference Frames

You probably found that you needed:
A starting point (an origin)
A set of directions (for instance left-right, forward-backward, up-down)
A unit of measure (to dictate how far to go in each direction)

Results - Reference Frames

In this course, we'll usually:
Define the origin as a location labeled "zero"
Create three perpendicular axes: x, y and z for direction
Use the meter as our unit of measure

The Axis

In this course, we will be solving problems in one-dimension.
Typically, we use the x-axis for that direction.
+x will usually be to the right
-x would then be to the left

We could define it the opposite way, but unless specified otherwise, this is what we'll assume. We also can think about compass directions in terms of positive and negative. For example, North would be positive and South negative.
The symbol for position is "x".

8 All of the following are examples of positive direction except:
A to the right
B north
C west
D up

9 All of the following are examples of negative direction except:
A to the right
B south
C east
D down
Displacement

Now that we understand how to define position, we can talk about a change in position; a displacement.

The symbol for "change" is the Greek letter "delta" "Δ".

So "Δx" means the change in x or the change in position.

Displacement describes how far you are from where you started, regardless of how you got there.

For instance, if you drive 60 miles from Pennsylvania to New Jersey...

and then 20 miles back toward Pennsylvania.

You have traveled:

a distance of 80 miles, and
a displacement of 40 miles,

since that is how far you are from where you started.

we can calculate displacement with the following formula:

\[
Δx = x_f - x_i
\]
Displacement

Measurements of distance can only be positive values (magnitudes) since it is impossible to travel a negative distance.

Imagine trying to measure a negative length with a meter stick...

Vectors and Scalars

Scalar - a quantity that has only a magnitude (number or value)
Vector - a quantity that has both a magnitude and a direction

Which of the following are vectors? Scalars?

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Vector</th>
<th>Scalar</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Distance</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Displacement</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Speed</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

10 How far your ending point is from your starting point is known as:

A distance
B displacement
C a positive integer
D a negative integer

11 A car travels 60m to the right and then 30m to the left. What distance has the car traveled?

12 You travel 60m to the right and then 30m to the left. What is the magnitude (and direction) of our displacement?
13 Starting from the origin, a car travels 4km east and then 7 km west. What is the total distance traveled?

A 3 km
B -3 km
C 7 km
D 11 km

14 Starting from the origin, a car travels 4km east and then 7 km west. What is the net displacement from the original point?

A 3 km west
B 3 km east
C 7 km west
D 11 km east

15 You run around a 400m track. At the end of your run, what is the distance that you traveled?

16 You run around a 400m track. At the end of your run, what is your displacement?

---

**Average Velocity**

Speed is defined as the ratio of distance and time

\[
s = \frac{d}{t}
\]

Similarly, velocity is defined as the ratio of displacement and time

\[
\bar{v} = \frac{\Delta x}{\Delta t}
\]
Average Velocity

Speeds are always positive, since speed is the ratio of distance and time; both of which are always positive.

\[ s = \frac{d}{t} \]

But velocity can be positive or negative, since velocity is the ratio of displacement and time; and displacement can be negative or positive.

\[ v = \frac{\Delta x}{\Delta t} \]

Usually, right is positive and left is negative.

17 Which of the following is a vector quantity?

A. time
B. velocity
C. distance
D. speed

18 Average velocity is defined as change in _____ over a period of ______.

A. distance, time
B. distance, space
C. displacement, time
D. displacement, space

19 You travel 60 meters to the right in 20 s; what is your average velocity?

20 An elephant travels 60 meters to the left in 20 s; what is the average velocity?

21 You travel 60 meters to the left in 20 s and then you travel 60 meters to the right in 30 s; what is your average velocity?
22. You travel 60 meters to the left in 20 s and then you travel 60 meters to the right in 30 s; what is your average speed?

23. You run completely around a 400 m track in 80 s. What was your average speed?

24. You run completely around a 400 m track in 80 s. What was your average velocity?

25. You travel 160 meters in 60 s; what is your average speed?

--

**Acceleration**

Acceleration is the rate of change of velocity.

\[ a = \frac{\Delta v}{\Delta t} \quad a = \frac{v - v_o}{t} \]

acceleration = \frac{\text{change of velocity}}{\text{elapsed time}}
Acceleration

\[ a = \frac{v - v_o}{t} \]

Acceleration is a vector, although in one-dimensional motion we only need the sign.

Since only constant acceleration will be considered in this course, there is no need to differentiate between average and instantaneous acceleration.

Units for Acceleration

Units for acceleration

You can derive the units by substituting the correct units into the right hand side of these equations.

\[ a = \frac{\Delta v}{\Delta t} \quad \text{m/s} = \text{m/s}^2 \]

26 Acceleration is the rate of change of ________.

A displacement
B distance
C speed
D velocity

27 The unit for velocity is:

A m
B m/s
C m/s²
D ft/s²

28 The metric unit for acceleration is:

A m
B m/s
C m/s²
D ft/s²

29 A horse gallops with a constant acceleration of 3m/s². Which statement below is true?

A The horse’s velocity doesn’t change.
B The horse moves 3m every second.
C The horse’s velocity increases 3m every second.
D The horse’s velocity increases 3m/s every second.
Solving Problems

After you read the problem carefully:

1. Draw a diagram (include coordinate axes).
2. List the given information.
3. Identify the unknown (what is the question asking?)
4. Choose a formula (or formulas to combine)
5. Rearrange the equations to isolate the unknown variable.
6. Substitute the values and solve!
7. Check your work.
   (You can do the same operations to the units to check your work ... dit!)

31 Your velocity changes from 60 m/s to the right to 20 m/s to the right in 20 s; what is your average acceleration?

32 Your velocity changes from 50 m/s to the left to 10 m/s to the right in 15 s; what is your average acceleration?

33 Your velocity changes from 90 m/s to the right to 20 m/s to the right in 5.0 s; what is your average acceleration?

Kinematics Equation 1
Motion at Constant Acceleration

\[ a = \frac{\Delta v}{\Delta t} \]

but since \( \Delta \) means change

\[ \Delta v = v - v_0 \]

and

\[ \Delta t = t - t_0 \]

if we always let \( t_0 = 0 \), \( \Delta t = t \)

\[ a = \frac{v - v_0}{t} \]

Solving for \( v \)

\[ v = v_0 + at \]

This equation tells us how an object's velocity changes as a function of time.

34 Starting from rest, you accelerate at 4.0 m/s\(^2\) for 6.0s. What is your final velocity?

35 Starting from rest, you accelerate at 8.0 m/s\(^2\) for 9.0s. What is your final velocity?

36 You have an initial velocity of 5.0 m/s. You then experience an acceleration of -1.5 m/s\(^2\) for 4.0s; what is your final velocity?

37 You have an initial velocity of -3.0 m/s. You then experience an acceleration of 2.5 m/s\(^2\) for 9.0s; what is your final velocity?

38 How much time does it take to accelerate from an initial velocity of 20m/s to a final velocity of 100m/s if your acceleration is 1.5 m/s\(^2\)?
39 How much time does it take to come to rest if your initial velocity is 5.0 m/s and your acceleration is -2.0 m/s²?

40 An object accelerates at a rate of 3 m/s² for 6 s until it reaches a velocity of 20 m/s. What was its initial velocity?

41 An object accelerates at a rate of 1.5 m/s² for 4 s until it reaches a velocity of 10 m/s. What was its initial velocity?

Graphing Motion at Constant Acceleration

In physics there is another approach in addition to algebraic which is called graphical analysis. The formula \( v = v_0 + at \) can be interpreted by the graph. We just need to recall our memory from math classes where we already saw a similar formula \( y = mx + b \).

From these two formulas we can some analogies:

\[ v = y \] (dependent variable of \( t \)),
\[ v_0 = b \] (intersection with vertical axis),
\[ t = x \] (independent variable),
\[ a = m \] (slope of the graph - the ratio between rise and run \( y/\Delta x \)).

Motion at Constant Acceleration

Below we can find the geometric explanation to the acceleration \( a = \Delta v/\Delta t \).

If slope is equal to: \( m = \Delta y/\Delta x \)

Then consider a graph with velocity on the \( y \)-axis and time on the \( x \)-axis. What is the slope for the graph on the right?

\[
\begin{align*}
\text{(slope)} & \quad \Rightarrow \quad y = \Delta y/\Delta x \\
\text{(slope of velocity vs. time)} & \quad \Rightarrow \quad a = \Delta v/\Delta t
\end{align*}
\]
Motion at Constant Acceleration

The acceleration graph as a function of time can be used to find the velocity of a moving object. When the acceleration is constant the velocity is changing by the same amount each second. This can be shown on the graph as a straight horizontal line.

In order to find the change in velocity for a certain limit of time we need to calculate the area under the acceleration line that is limited by the time interval.

Free Fall: Acceleration Due to Gravity

Free Fall

All unsupported objects fall towards Earth with the same acceleration. We call this acceleration the “acceleration due to gravity” and it is denoted by g.

\[ g = 9.8 \, \text{m/s}^2 \]

Keep in mind, ALL objects accelerate towards the earth at the same rate.

\[ g \] is a constant!
A ball is dropped from rest and falls (do not consider air resistance). Which is true about its motion?

- A acceleration is constant
- B velocity is constant
- C velocity is decreasing
- D acceleration is decreasing

An acorn falls from an oak tree. You note that it takes 2.5 seconds to hit the ground. How fast was it going when it hit the ground?
47 A rock falls off a cliff and hits the ground 5 seconds later. What velocity did it hit the ground with?

48 A ball is thrown down off a bridge with a velocity of 5 m/s. What is its velocity 2 seconds later?

49 An arrow is fired into the air and it reaches its highest point 3 seconds later. What was its velocity when it was fired?

50 A rocket is fired straight up from the ground. It returns to the ground 10 seconds later. What was its launch speed?

51 Starting from rest you accelerate to 20 m/s in 4.0s. What is your average velocity?

Motion at Constant Acceleration

If velocity is changing at a constant rate, the average velocity is just the average of the initial and final velocities.

\[ \bar{v} = \frac{v + v_i}{2} \]

And we learned earlier that

\[ \bar{v} = \frac{\Delta x}{t} \]

\[ \frac{\Delta x}{t} = \frac{v + v_i}{2} \]

Some problems can be solved most easily by using these two equations together.
52. Starting with a velocity of 12 m/s you accelerate to 48 m/s in 6.0 s. What is your average velocity?

53. Starting with a velocity of 12 m/s you accelerate to 48 m/s in 6.0 s. Using your previous answer, how far did you travel in that 6.0 s?

\[ v_{\text{avg}} = \Delta x / t \]
\[ \Delta x = v_{\text{avg}} \times t = (v_f + v_i) / 2 \times t \]
\[ v_{\text{avg}} = 30 \text{ m/s} / (6 \text{ s}) \]
\[ v_{\text{avg}} = 180 \text{ m/s} \]

54. A car accelerates from rest to 30 m/s while traveling a distance of 20 m; what was its acceleration?

55. An object accelerates from rest, with a constant acceleration of 8.4 m/s², what will its velocity be after 11 s?

Position vs Time Graphs

An object’s position at any point in time can be graphed.

These graphs show position but also can be used to find an object’s velocity.

Position is the dependent variable (y-axis), and time is the independent variable (x-axis).
Creating a Position vs. Time Graph

1. Draw a cartesian coordinate system by drawing a vertical and horizontal axis.

2. Label the vertical axis as position (x), and the horizontal axis as time (t).

3. Add units next to each axis label, showing position measured in meters, and time measured in seconds.

4. Add points to the graph requires both the position and time it happened.

Starting at the position, \( x_0 = 4 \text{ m} \), you travel at a constant velocity of \( +2 \text{ m/s} \) for 6s.

a. Determine your position at the times of 0s; 2s; 5s; and 6s.

Starting at the position, \( x_0 = 4 \text{ m} \), you travel at a constant velocity of \( +1 \text{ m/s} \) for 6s.

b. Draw the Position versus Time for your travel during this time.

Starting at the position, \( x_0 = 4 \text{ m} \), you travel at a constant velocity of \( +2 \text{ m/s} \) for 6s.

c. Draw the Velocity versus Time graph for your trip.

Starting at the position, \( x_0 = 10 \text{ m} \), you travel at a constant velocity of \( -1 \text{ m/s} \) for 6s.

b. Draw the Position versus Time for your travel during this time.

Velocity vs. Time Graphs

Similarly, the same approach can be used to create a velocity vs. time graph.

A velocity versus time graph differs by having the velocity on the vertical axis.

A velocity versus time graph shows describes an object's velocity, it's displacement, and acceleration.
Analyzing Position vs Time Graphs

Recall earlier in this unit that slope was used to describe motion.

The slope in a position vs. time graph is $\Delta x/\Delta t$, which is equal to velocity.

Therefore, slope is equal to velocity on a position vs. time graph.

A positive slope is a positive velocity, a negative slope is a negative velocity, and a slope of zero means zero velocity.

A positive velocity means moving in the positive direction, a negative velocity means moving in the negative direction, and zero velocity means not moving at all.

The position versus time graph, below, describes the motion of three different cars moving along the x-axis.

a. Describe, in words, the velocity of each of the cars. Make sure you discuss each car’s speed and direction.

b. Calculate the velocity of each of the cars.

c. Draw, on one set of axes, the Velocity versus Time graph for each of the three cars.

The velocity vs time graph, below, describes the motion of an object moving along the x-axis.

Describe in words what is happening to the speed during the following intervals.

a) 0s to 1s  b) 1s to 3s  c) 3s to 4 sec  d) 4s to 5s  e) 5s to 6s
The velocity vs time graph, below, describes the motion of an object moving along the x-axis.

Determine the average speed during the following intervals.
- a) 0s to 1s
- b) 1s to 3s
- c) 3s to 4 sec
- d) 4s to 5s
- e) 5s to 6s
- f) 4s to 6s

The velocity vs time graph, below, describes the motion of an object moving along the x-axis.

Determine the displacement during the following intervals.
- a) 0s to 1s
- b) 1s to 3s
- c) 3s to 4 sec
- d) 4s to 5s
- e) 5s to 6s
- f) 3s to 5s

56 Determine the net displacement during the first four seconds of travel.

Summary

- **Kinematics** is the description of how objects move with respect to a defined reference frame.
- **Displacement** is the change in position of an object.
- **Average velocity** is the displacement divided by the time.
- **Average acceleration** is the change in velocity divided by the time.